O K L A H O M A S T A T E U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 4503
Random Signals and Noise
Spring 2007

Midterm Exam #1

PLEASE DO ALL FIVE PROBLEMS

Name : _____

E-Mail Address:_____

Problem 1:
At a military installation, six similar radars are placed in operation. It is known that a radar's probability of failing to operate before 500 hours of "on" time have accumulated is 0.06. What are the probability that before 500 hours have elapsed, (a) all will operate, (b) all will fail, and (c) only one will fail?

Problem 2:

The lifetime of a system expressed in weeks is a Rayleigh random variable X for which

$$f_X(x) = \begin{cases} (x/200)e^{-x^2/400}, & x \geq 0 \\ 0, & x < 0 \end{cases}.$$

(a) What is the probability that the system will not last a full week?

(b) What is the probability the system lifetime will exceed one week?

Problem 3:

Please show conditional distribution and density functions for general event $B = \{a < X \le b\}$ are

$$F_X(x|a < X \le b) = \begin{cases} 0, & x < a \\ \dfrac{F_X(x) - F_X(a)}{F_X(b) - F_X(a)}, & a \le x < b \\ 1, & x \ge b \end{cases}, \text{ and}$$

$$f_X(x|a < x \le b) = \begin{cases} 0, & x < a \\ \dfrac{f_X(x)}{F_X(b) - F_X(a)}, & a \le x < b \\ 0, & x \ge b \end{cases}.$$

Problem 4:
Suppose it is found that the density function

$$f_X(x) = \frac{16/\pi}{(4+x^2)^2}$$

is a good empirical fit to the probability density function of some random experiment data represented by a random variable *X*. Find the mean and variance of *X*.

Problem 5:
Show that the second moment of any random variable *X* about an arbitrary point a is minimum when $a = \overline{X}$; that is, show that $E\left[(X-a)^2\right]$ is mimimum for $a = \overline{X}$.